

Melting of antikaon condensate in protoneutron stars

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Abstract

We study the melting of a K^- condensate in hot and neutrino-trapped protoneutron stars. In this connection, we adopt relativistic field theoretical models to describe the hadronic and condensed phases. It is observed that the critical temperature of antikaon condensation is enhanced as baryon density increases. For a fixed baryon density, the critical temperature of antikaon condensation in a protoneutron star is smaller than that of a neutron star. We also exhibit the phase diagram of a protoneutron star with a K^- condensate.

PACS numbers: 26.60.+c, 21.65.+f, 97.60.Jd, 95.30.Cq

I. INTRODUCTION

Recent observation of a $2M_{\odot}$ neutron star puts stringent conditions on the equation of state (EoS) of dense matter in neutron star interior [1]. It has been long conjectured that neutron star interior might contain exotic phases of dense matter such as hyperons, Bose-Einstein condensates of antikaons and quarks. In certain cases, exotic components of matter make the EoS softer resulting in maximum masses of neutron stars below $2M_{\odot}$. For example, the equations of state involving hyperons lead to maximum neutron star mass below $2M_{\odot}$ when the hyperon-scalar meson coupling is determined from existing hypernuclei data and hyperon-vector meson couplings are estimated from $SU(6)$ quark model [2, 3]. Recently, it has been demonstrated that the neutron star mass of $2M_{\odot}$ could be achieved by making hyperon-vector meson coupling stronger [4–6]. On the other hand, it was shown that the EoS including antikaon condensates might result in $2M_{\odot}$ neutron stars if the magnitude of the attractive antikaon optical potential depth is 140 MeV or less [7].

The idea of antikaon condensation in dense baryonic matter formed in heavy ion collisions as well as in neutron stars, started with the seminal work of Kaplan and Nelson [8]. They investigated the problem within the $SU(3)_L \times SU(3)_R$ chiral perturbation theory. Later detailed studies on antikaon condensation in neutron star interior were carried out in the chiral perturbation theory [9–13] as well as meson exchange models [2, 7, 14–21]. The first order phase transition from nuclear matter to antikaon condensed matter was either studied using Maxwell construction or Gibbs’ phase equilibrium rules. It was observed that the threshold density for K^- condensation was sensitive to the nuclear EoS as well as the strength of the attractive antikaon optical potential depth.

Here we are interested in the melting of antikaon condensate in newly born hot and neutrino-trapped protoneutron stars. The study of antikaon condensation continued at finite temperatures in connection with the metastability of protoneutron stars [22] as well as the dynamical evolution of the condensation [23]. The critical temperature of antikaon condensation in hot neutron stars, just after emission of trapped neutrinos, was investigated for the first time [24]. We have recently investigated the thermal nucleation of droplets of antikaon condensed matter in neutron stars after neutrinos were emitted as well as in protoneutron stars [25, 26]. Further we made use of the results of our earlier calculation on the critical temperature of antikaon condensation in the thermal nucleation of antikaon

droplets in neutron stars [24, 25]. However, there are no calculations on the critical temperature of antikaon condensation in hot and neutrino-trapped protoneutron stars. This might have important implications to understand whether the droplet of antikaon condensed phase would melt or not during the thermal nucleation in protoneutron stars. This motivates us to investigate the critical temperature of K^- condensation in protoneutron stars.

The organisation of the paper is the following. We discuss the model to compute the composition and EoS involving K^- condensate at finite temperature in Section II. Results are discussed in Section III. Section IV gives a summary.

II. FORMALISM

We investigate the melting of a K^- condensate in hot and neutrino trapped protoneutron stars. We adopt the finite temperature calculations of Ref.[22, 24] to study this problem. The antikaon condensation is treated as a second order phase transition from hadronic to K^- condensed matter in protoneutron stars. Both phases of matter are made of neutrons (n), protons (p), electrons, neutrinos, thermal K^- mesons; K^- condensate only in the latter phase. Both phases maintain local charge neutrality and beta-equilibrium conditions. We can write down the total thermodynamical potential of both phases as

$$\Omega_{tot} = \Omega_N + \Omega_K + \Omega_L . \quad (1)$$

We describe the hadronic phase using a relativistic field theoretical model where baryons are interacting by the exchange of σ , ω and ρ mesons. The corresponding Lagrangian density is given by [27, 28],

$$\begin{aligned} \mathcal{L}_B = & \sum_{B=n,p} \bar{\Psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - g_{\rho B} \gamma_\mu \mathbf{t}_B \cdot \boldsymbol{\rho}^\mu) \Psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu , \end{aligned} \quad (2)$$

where ψ_B denotes the Dirac bispinor for baryons B , m_B is the vacuum mass and the isospin operator is \mathbf{t}_B . The scalar self-interaction term [28] is

$$U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 . \quad (3)$$

The thermodynamic potential per unit volume of the hadronic phase is given by [29],

$$\begin{aligned} \frac{\Omega_N}{V} = & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & - 2T \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} [\ln(1 + e^{-\beta(E^* - \nu_i)}) + \ln(1 + e^{-\beta(E^* + \nu_i)})] , \end{aligned} \quad (4)$$

where the temperature is related to $\beta = 1/T$, $E^* = \sqrt{(k^2 + m_N^{*2})}$ and the effective baryon mass $m_N^* = m_N - g_{\sigma N}\sigma$. Neutron and proton chemical potentials are obtained from $\mu_n = \nu_n + g_{\omega N}\omega_0 - \frac{1}{2}g_{\rho N}\rho_{03}$ and $\mu_p = \nu_p + g_{\omega N}\omega_0 + \frac{1}{2}g_{\rho N}\rho_{03}$.

We can immediately calculate the thermodynamic quantities of the hadronic phase such as the pressure $P_N = -\Omega_N/V$ and the energy density

$$\begin{aligned} \epsilon_N = & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & + 2 \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} E^* \left(\frac{1}{e^{\beta(E^* - \nu_i)} + 1} + \frac{1}{e^{\beta(E^* + \nu_i)} + 1} \right) . \end{aligned} \quad (5)$$

Similarly we can compute neutron and proton number densities which include contributions from both particle and antiparticles [22, 24].

Next we describe the antikaon condensed phase. The thermodynamic potential of this phase is calculated adopting the finite temperature treatment of antikaon condensation by Pons et al. [22]. In this case, we treat the (anti)kaon-baryon interaction in the same footing as the baryon-baryon interaction. The Lagrangian density for (anti)kaons in the minimal coupling scheme is [14, 19],

$$\mathcal{L}_K = D_\mu^* \bar{K} D^\mu K - m_K^{*2} \bar{K} K , \quad (6)$$

where the covariant derivative is $D_\mu = \partial_\mu + ig_{\omega K}\omega_\mu + ig_{\rho K}\mathbf{t}_K \cdot \boldsymbol{\rho}_\mu$ and the effective mass of (anti)kaons is $m_K^* = m_K - g_{\sigma K}\sigma$.

The thermodynamic potential per unit volume in the antikaon condensed phase is [22]

$$\frac{\Omega_K}{V} = T \int \frac{d^3p}{(2\pi)^3} [\ln(1 - e^{-\beta(\omega_{K^-} - \mu)}) + \ln(1 - e^{-\beta(\omega_{K^+} + \mu)})] , \quad (7)$$

where the in-medium energies of K^\pm mesons are given by

$$\omega_{K^\pm} = \sqrt{(p^2 + m_K^{*2})} \pm \left(g_{\omega K}\omega_0 + \frac{1}{2}g_{\rho K}\rho_{03} \right) , \quad (8)$$

and the chemical potential of K^- mesons is $\mu = \mu_n - \mu_p$. The threshold condition for s -wave K^- condensation is given by $\mu = \omega_{K^-} = m_K^* - g_{\omega K}\omega_0 - \frac{1}{2}g_{\rho K}\rho_{03}$.

The number density of (anti)kaons is given by

$$n_K = n_K^C + n_K^T, \quad (9)$$

which has contributions from the condensate n_K^C and thermal (anti)kaons given by,

$$n_K^T = \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{e^{\beta(\omega_{K^-} - \mu)} - 1} - \frac{1}{e^{\beta(\omega_{K^+} + \mu)} - 1} \right). \quad (10)$$

In the antikaon condensed phase, only thermal (anti)kaons contribute to the pressure $P_K = -\Omega_K/V$; the condensate does not contribute to the pressure. The energy density of (anti)kaons is given by

$$\epsilon_K = m_K^* n_K^C + \left(g_{\omega K} \omega_0 + \frac{1}{2} g_{\rho K} \rho_0 \right) n_K^T + \int \frac{d^3p}{(2\pi)^3} \left(\frac{\omega_{K^-}}{e^{\beta(\omega_{K^-} - \mu)} - 1} + \frac{\omega_{K^+}}{e^{\beta(\omega_{K^+} + \mu)} - 1} \right). \quad (11)$$

Finally, we can calculate thermodynamic quantities of electrons, neutrinos and their antiparticles from the thermodynamic potential per unit volume

$$\frac{\Omega_L}{V} = -T \sum_l g_l \int \frac{d^3k}{(2\pi)^3} [\ln(1 + e^{-\beta(E_l - \mu_l)}) + \ln(1 + e^{-\beta(E_l + \mu_l)})], \quad (12)$$

where $g_l=2$ for electrons and 1 for neutrinos.

We use mean field approximations in this calculation. Mean meson fields are obtained by minimising the total thermodynamic potential of Eq. (1). The total energy density in the condensed phase is $\epsilon = \epsilon_N + \epsilon_K + \epsilon_L$. Similarly the total entropy per baryon is given by $S = (\mathcal{S}_N + \mathcal{S}_K + \mathcal{S}_L)/n_b$, where \mathcal{S}_N , \mathcal{S}_K and \mathcal{S}_L are entropy densities of the hadronic phase, antikaon condensed phase and leptons, respectively [22, 24].

Further the EoS of hot and lepton-trapped matter is constrained by the charge neutrality and β equilibrium conditions which are given by,

$$n_p - n_K - n_e = 0, \quad (13)$$

$$\mu = \mu_n - \mu_p = \mu_e - \mu_{\nu_e}. \quad (14)$$

III. RESULTS AND DISCUSSION

We use the GM1 parameter set for nucleon-meson coupling constants which reproduce the nuclear matter saturation properties i.e. binding energy -16 MeV, saturation density (n_0) 0.153 fm^{-3} , asymmetry energy coefficient 32.5 MeV, effective nucleon mass (m_N^*/m_n) 0.70 and incompressibility $K = 300 \text{ MeV}$ [19, 30].

Kaon-vector meson couplings are obtained from the quark model and isospin counting rules [14, 24]. Further the kaon-scalar meson coupling is estimated from the real part of antikaon optical potential depth at normal nuclear matter density [7, 14, 19]. It was already noted that the value of antikaon optical potential depth varied widely from $U_K = -180 \pm 20$ MeV as obtained from the analysis of kaonic atom data [31–33] to ~ -60 MeV in the chiral model [34]. Earlier we observed that the EoS involving the K^- condensate became very soft when the antikaon potential was highly attractive. Such an EoS resulted in maximum neutron star mass below $2M_\odot$ [7]. After the discovery of $2M_\odot$ neutron star, the EoS including exotic matter is severely constrained. Therefore we perform this calculation for $U_K = -120$ MeV which results in a maximum neutron star mass of $2.08M_\odot$ [7]. Kaon-scalar coupling constants are taken from Table II of Ref. [19].

We consider a set of values for entropy per baryon S in this calculation. Further we take lepton fraction $Y_L = Y_e + Y_{\nu_e} = 0.35$ in the protoneutron star. These correspond to several snapshots in the evolution of neutrino-trapped and hot protoneutron stars. For a fixed entropy per baryon, the temperature varies from the center to the surface in the protoneutron star. This is demonstrated in Figure 1. We highlight this for entropy per baryon $S = 2$. The temperature increases with baryon density in this case. Here, we also show the corresponding scenario in a hot and neutrino-free neutron star for $S = 2$. Higher temperature is obtained in the neutron star. Further we note that as soon as the antikaon condensate appears in the protoneutron star, there is a drop in the temperature compared with the case without the condensate.

Populations of different particle species in the β -equilibrated protoneutron star matter are shown with baryon density for $S = 2$ case in Figure 2. We observe that the matter is populated with thermal kaons well before the onset of the K^- condensate. This also leads to the enhancement of the proton fraction. The threshold density of K^- condensation in the hot and neutrino-trapped protoneutron star matter for entropy per baryon $S = 2$ is $4.2n_0$. On other hand, K^- condensation in the neutrino-trapped matter at zero temperature sets in at $3.68n_0$. Similarly, threshold densities of antikaon condensation in neutrino-free neutron stars with $U_K = -120$ MeV are $3.16n_0$ for $S = 2$ and $3.05n_0$ for $S = 0$. The role of finite temperature is to shift the threshold of antikaon condensation to higher density. After the appearance of the condensate, K^- density (n_K^C) increases rapidly resulting in higher proton number density in the protoneutron star as evident from Fig. 2.

We discuss the thermal effects on the EoS and protoneutron star masses. Pressure versus energy density is plotted for entropy per baryon $S = 2$ and $S = 0$ in Figure 3. We do not find any appreciable change in the EoS due to thermal effects. This result has important significance for the calculation of thermal nucleation of the antikaon condensed phase in protoneutron stars and justifies the use of a zero temperature EoS in that case [26]. Maximum protoneutron star masses for $S = 2$ and $S = 0$ cases are 2.228 and $2.214M_\odot$, respectively. The corresponding central density for $S = 2$ case is $5.3n_0$ and it is $5.59n_0$ for $S = 0$.

Next we investigate the melting of the condensate in protoneutron star matter as the interior temperature increases. As we follow the evolution of a protoneutron star through several snapshots, we consider different values of entropy per baryon. We demonstrate the variation of entropy per baryon with temperature for several fixed values of baryon densities and $U_K = -120$ MeV in Figure 4. The condensate density (n_K^C) is a function of both baryon density and temperature. The condensate disappears above a certain temperature known as critical temperature (T_C). To obtain this critical temperature, we show the ratio of the condensate density ($n_K^C(T)$) at finite temperature to that ($n_K^C(T = 0)$) of zero temperature as a function of temperature for several fixed baryon densities and $U_K = -120$ MeV in Figure 5. In each case corresponding to a fixed baryon density, the condensate density diminishes with increasing temperature resulting ultimately in the meltdown of the condensate at the critical temperature of the condensation. We find that the critical temperature increases as baryon density increases.

In an earlier calculation, we estimated the critical temperature of K^- condensation for hot and neutrino-free neutrons stars [24]. When we compare the critical temperatures in both cases for a certain baryon density for example $4.4n_0$, the critical temperature in the protoneutron star has a smaller value than that of the neutron star as evident from Figure 6. This may be attributed to more heat content in the protoneutron star than that of the neutron star. For example, at temperature $T = 40$ MeV, entropy per baryon corresponding to $Y_L = 0.35$ and $Y_{\nu_e} = 0$ are 1.6 and 1.3, respectively. This finding might be very useful in understanding the thermal nucleation of droplets of antikaon condensed matter in protoneutron stars [26].

As we know the critical temperature as a function of baryon density, we can construct a phase diagram of protoneutron star matter involving a K^- condensate. The phase diagram as temperature versus baryon density is shown in Figure 7. The solid line representing critical

temperatures separates the condensed phase (shaded region) from the hadronic phase.

IV. SUMMARY

We have investigated the critical temperature of K^- condensation in β equilibrated hot and neutrino-trapped protoneutron stars for antikaon optical potential depth $U_K = -120$ MeV within the framework of field theoretical models at finite temperature. The critical temperature of antikaon condensation in hot and neutrino-trapped protoneutron stars increases with baryon density. It is also noted that the critical temperature of antikaon condensation in a protoneutron star is smaller than that of a hot and neutrino-free neutron star. This result might play an important role in the thermal nucleation of droplets of antikaon condensed matter in protoneutron stars.

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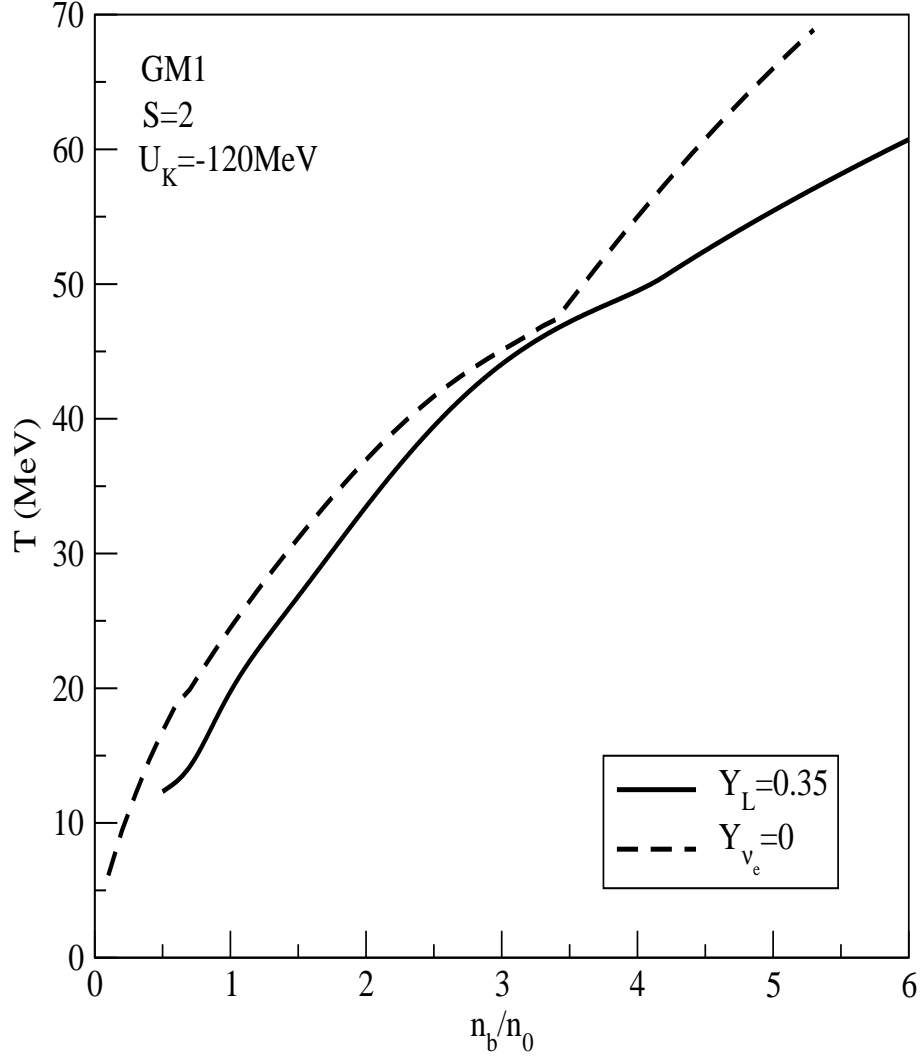


FIG. 1. Temperature is plotted with normalised baryon density for entropy per baryon $S = 2$, $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120 \text{ MeV}$.

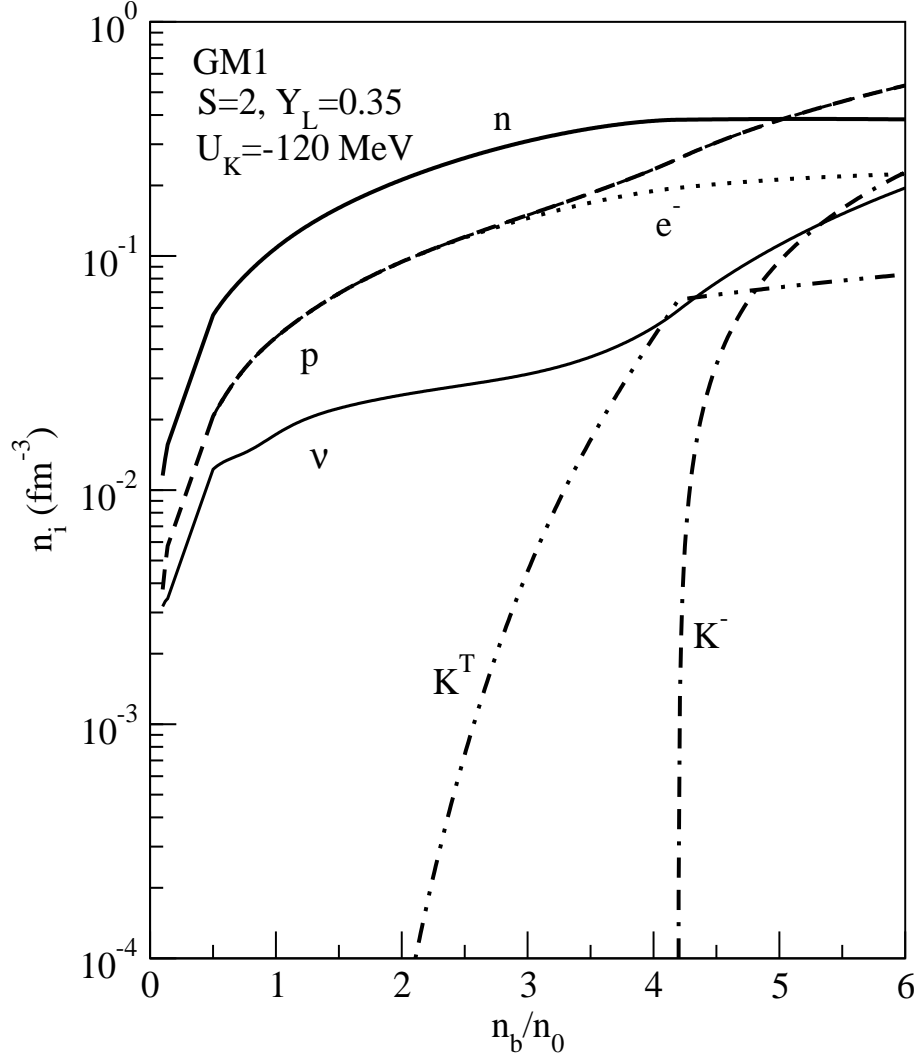


FIG. 2. Populations of different particle species in β -equilibrated hot and lepton-trapped matter including a K^- condensate are shown as a function of normalised baryon density for entropy per baryon $S = 2$, $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120$ MeV.

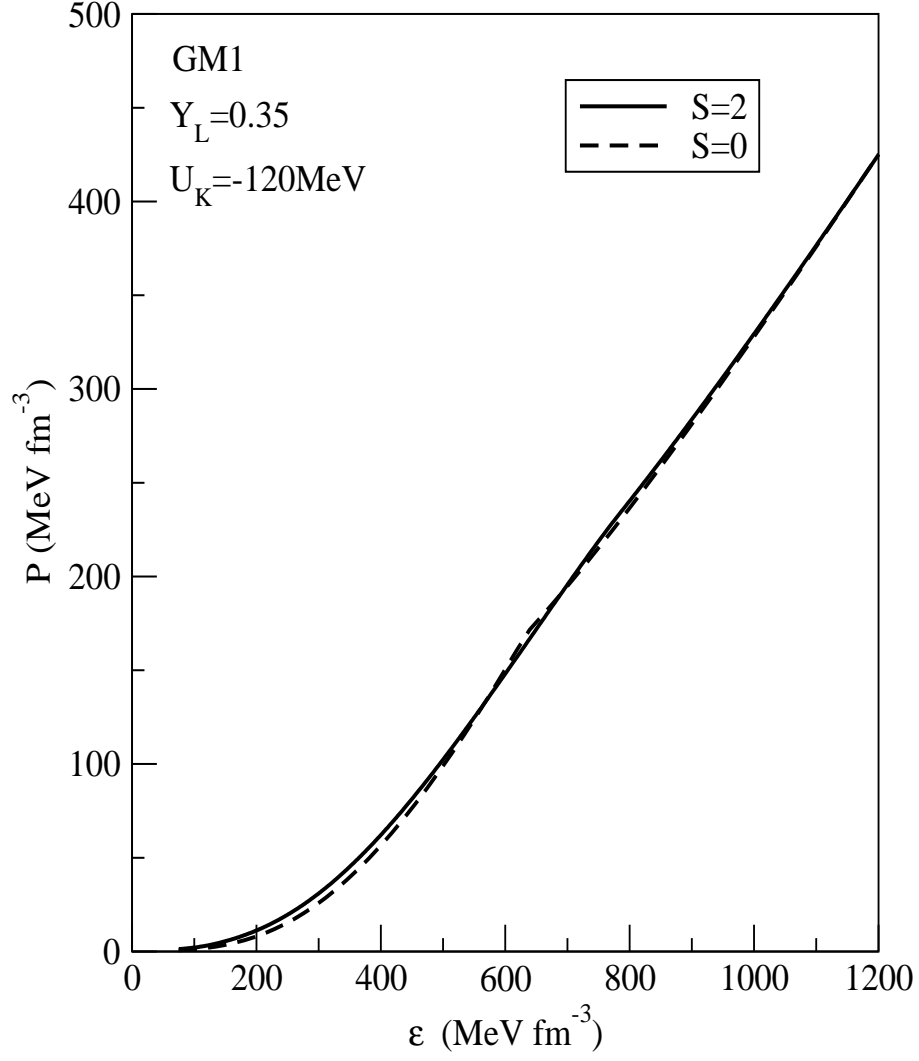


FIG. 3. Pressure is plotted with energy density for entropy per baryon $S = 2$ and $S = 0$ and $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120$ MeV.

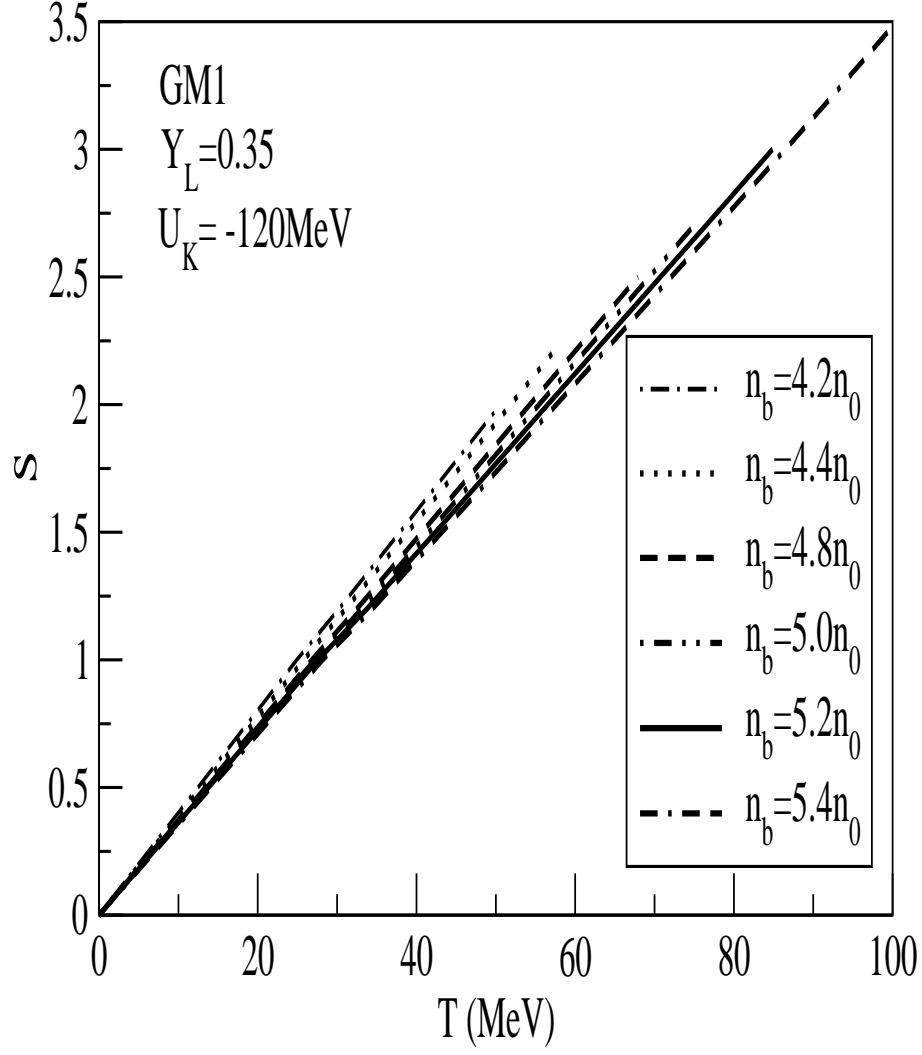


FIG. 4. Entropy per baryon is plotted with temperature for fixed baryon number densities, $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120$ MeV.

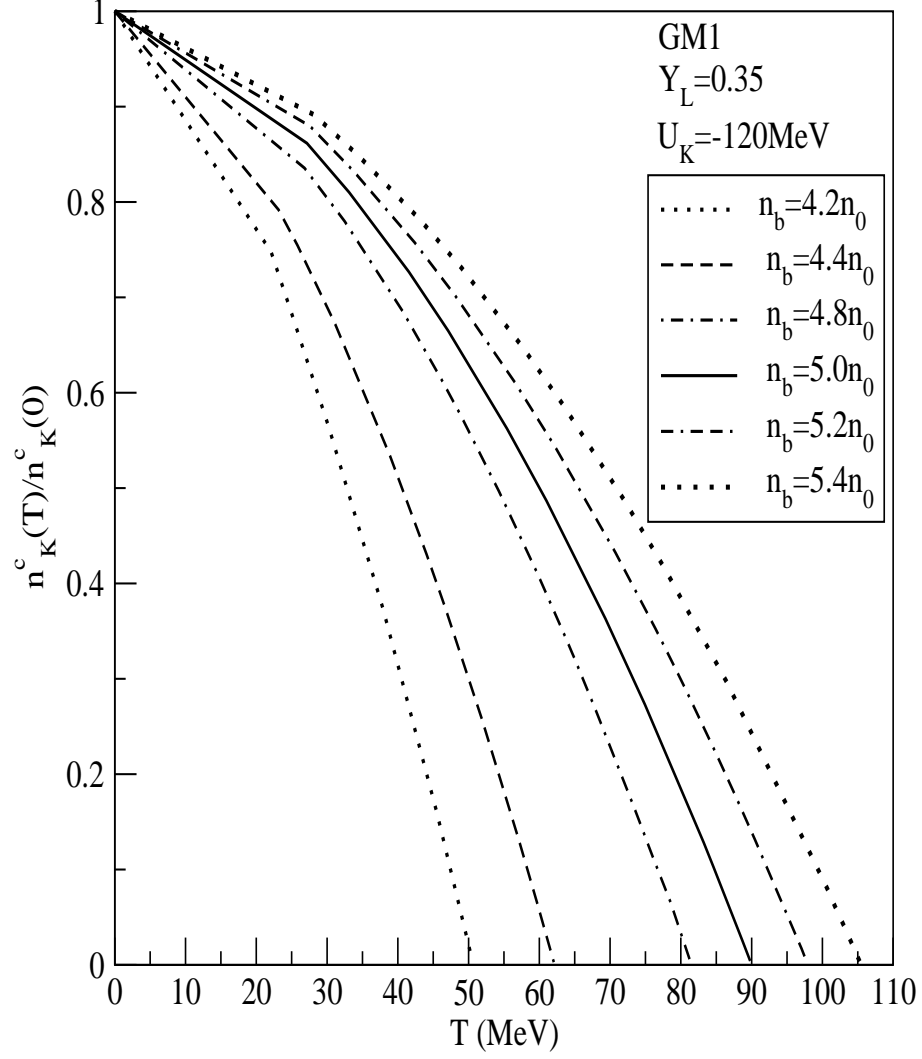


FIG. 5. The ratio of the condensate density at a nonzero temperature to that of zero temperature is plotted with temperature for fixed baryon number densities, $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120$ MeV.

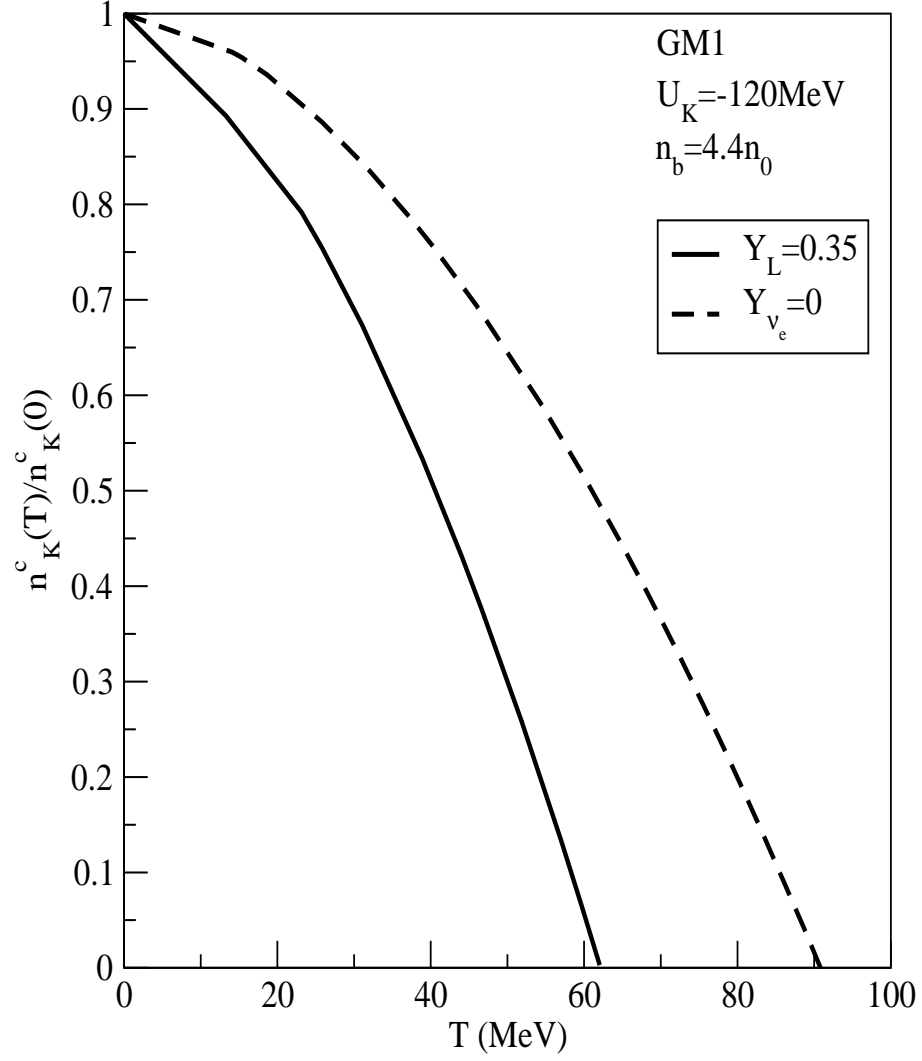


FIG. 6. The ratio of the condensate density at a nonzero temperature to that of zero temperature is plotted with temperature for a hot and neutrino-trapped protoneutron star as well as a hot and neutrino-free neutron star at a fixed baryon number density $4.4n_0$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120 \text{ MeV}$.

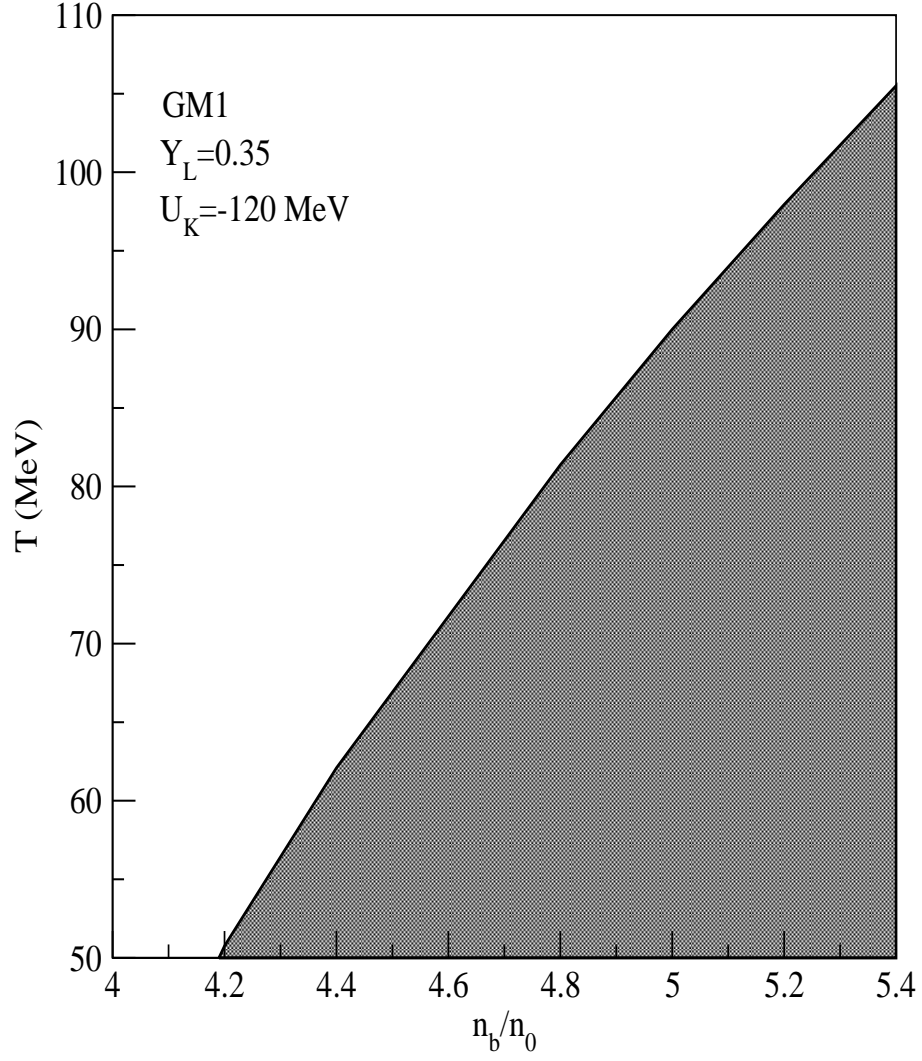


FIG. 7. Phase diagram of protoneutron star matter with K^- condensate. The solid line corresponds to critical temperatures of K^- condensation for $Y_L = 0.35$ and antikaon optical potential depth at normal nuclear matter density $U_K = -120$ MeV.